

Economics of monetary unions

Chapter 5. Fragility of monetary union

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The Model

I would like to thank Paul De Grauwe for providing his lecture notes.

```
In[1]:= ClearAll["Global`*"] (*completely clear global symbols to start fresh*)
```

• The Phillips curve :

$$U = U_N + a(\pi^e - \pi) + \varepsilon \quad (1)$$

```
In[2]:= U = UN + a (pe - p) + e
```

```
Out[2]= a (-p + pe) + UN + e
```

• Alternatively, you can specify the supply curve :

$$Y = Y_n + \theta(\pi - \pi^e) + \varepsilon \quad (2)$$

• Loss function of authorities:

$$L = \pi^2 + \beta(U - \bar{U})^2 \quad (3)$$

$$\text{In[3]:= } \mathbf{L = p^2 + \beta (U - UB)^2}$$

$$\text{Out[3]= } \mathbf{p^2 + \beta (a (-p + pe) - UB + UN + \epsilon)^2}$$

with UB = target unemployment

$$\bar{U} = \lambda U_N \quad (4)$$

$$\text{In[4]:= } \mathbf{UB = \lambda UN}$$

$$\text{Out[4]= } \mathbf{\lambda UN}$$

where $\lambda < 1$

In[*]:= L

In[5]:= $p^2 + \beta (a (-p + pe) + UN + \epsilon - \lambda UN)^2$

Out[5]= $p^2 + \beta (a (-p + pe) + UN + \epsilon - \lambda UN)^2$

$$L = \pi^2 + \beta [(1 - \lambda) U_N + a(\pi^e - \pi) + \epsilon]^2 \quad (5)$$

- Workers set wages based on pe
- Authorities minimize loss given these expectations
- Note : authorities directly control inflation

- Authorities minimize loss given these expectations

$$\frac{dL}{d\pi} = 2\pi + 2\beta(-a)[(1-\lambda)U_N + a(\pi^e - \pi) + \varepsilon] = 0 \quad (6)$$

In[6]:= **CBLoss = D[L, p]**

Out[6]= $2p - 2a\beta(a(-p + pe) + UN + \varepsilon - \lambda UN)$

In[7]:= **CBLoss1 = Simplify[Collect[CBLoss, \beta a] / 2]**

Out[7]= $p - a\beta(a(-p + pe) + UN + \varepsilon - \lambda UN)$

In[8]:= **Collect[CBLoss1, p]**

Out[8]= $-a^2 pe \beta - a UN \beta + p(1 + a^2 \beta) - a \beta \varepsilon + a \beta \lambda UN$

$$\pi(1 + \beta a^2) - \beta a(1 - \lambda)U_N - \beta a^2 \pi^e - \beta a \varepsilon = 0 \quad (7)$$

$$\pi(1 + \beta a^2) - \beta a(1 - \lambda) U_N - \beta a^2 \pi^e - \beta a \varepsilon = 0 \quad (8)$$

In[9]:= $\mathbf{p (1 + \beta a^2) - \beta a (1 - \lambda) UN - \beta a^2 pe - \beta a \varepsilon == 0}$

Out[9]= $-\mathbf{pe \beta a^2 + p (1 + \beta a^2) - \beta a \varepsilon - UN \beta a (1 - \lambda) == 0}$

In[10]:= $\mathbf{Solve[-pe \beta a^2 + p (1 + \beta a^2) - UN \beta a (1 - \lambda) - \beta a \varepsilon == 0, p]}$

Out[10]=

$$\left\{ \left\{ p \rightarrow \frac{UN \beta a + pe \beta a^2 + \beta a \varepsilon - UN \beta a \lambda}{1 + \beta a^2} \right\} \right\}$$

$$\pi = \frac{\beta a(1 - \lambda) U_N}{1 + \beta a^2} + \frac{\beta a^2 \pi^e}{1 + \beta a^2} + \frac{\beta a \varepsilon}{1 + \beta a^2} \quad (9)$$

This is optimal policy rule of authorities given expectations

Agents know this rule

Thus when forming expectations p_e they use this rule

$$\pi = \frac{\beta a(1-\lambda) U_N}{1+\beta a^2} + \frac{\beta a^2 \pi^e}{1+\beta a^2} + \frac{\beta a \varepsilon}{1+\beta a^2} \quad (10)$$

$$\text{In[11]:= } \mathbf{p} = \frac{\beta \mathbf{a} (1 - \lambda) \mathbf{UN} + p_e \beta \mathbf{a}^2 + \beta \mathbf{a} \varepsilon}{1 + \beta \mathbf{a}^2}$$

$$\text{Out[11]= } \frac{p_e \beta \mathbf{a}^2 + \beta \mathbf{a} \varepsilon + \mathbf{UN} \beta \mathbf{a} (1 - \lambda)}{1 + \beta \mathbf{a}^2}$$

1. Set $p = p_e$ and assume first no shocks ($\epsilon = 0$)

$$\pi = \frac{\beta a(1-\lambda) U_N}{1+\beta a^2} + \frac{\beta a^2 \pi^e}{1+\beta a^2} + \frac{\beta a \epsilon}{1+\beta a^2} \quad (11)$$

$$\pi^e = \frac{\beta a(1-\lambda) U_N}{1+\beta a^2} + \frac{\beta a^2 \pi^e}{1+\beta a^2} \quad (12)$$

In[12]:= `Solve` $\left[\frac{\beta a (1 - \lambda) U_N + p_e \beta a^2}{1 + \beta a^2} - p_e == 0, p_e \right]$

Out[12]=

`{ { p_e → -UN β a (-1 + λ) } }`

$$\pi = \pi^e = \beta a(1-\lambda) U_N \quad (13)$$

(*there is an inflation bias (non-zero inflation), which increases with β, a .)

When shocks (e . g . , deterioration of trade balance) occur, solution is

$$\pi = \pi^e + \frac{\beta a}{1+\beta a^2} \varepsilon = \beta a(1 - \lambda) U_N + \frac{\beta a}{1+\beta a^2} \varepsilon \quad (14)$$

Authorities react to shocks in a stabilising manner depending on β

- if $\beta = 0$: no stabilisation (strict inflation targeting)
- if $\beta > 0$: some stabilisation

Substitute optimal inflation $\pi^e - \pi$ in Phillips curve

$$\text{In[*]} := \text{UN} + a \left(-\frac{\beta a}{1 + \beta a^2} \epsilon \right) + \epsilon$$

Out[*]=

$$\text{UN} + \epsilon - \frac{a \beta a \epsilon}{1 + \beta a^2}$$

$$\text{In[*]} := \text{Together} \left[\text{UN} + \epsilon - \frac{a \beta a \epsilon}{1 + \beta a^2} \right]$$

Out[*]=

$$\frac{\text{UN} + \text{UN} \beta a^2 + \epsilon - a \beta a \epsilon + \beta a^2 \epsilon}{1 + \beta a^2}$$

$$\text{In[*]} := \text{FullSimplify} \left[\frac{\text{UN} + \text{UN} \beta a^2 + \epsilon - a \beta a \epsilon + \beta a^2 \epsilon}{1 + \beta a^2} \right]$$

Out[*]=

$$\text{UN} + \frac{(1 - a \beta a + \beta a^2) \epsilon}{1 + \beta a^2}$$

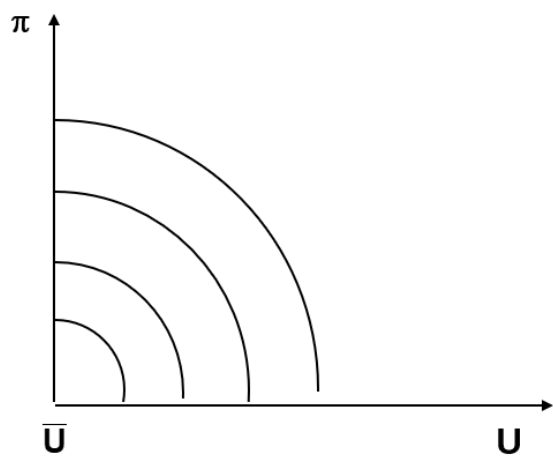
$$U = U_N + \frac{1}{1+\beta a^2} \varepsilon \quad (15)$$

If $\beta = 0$,

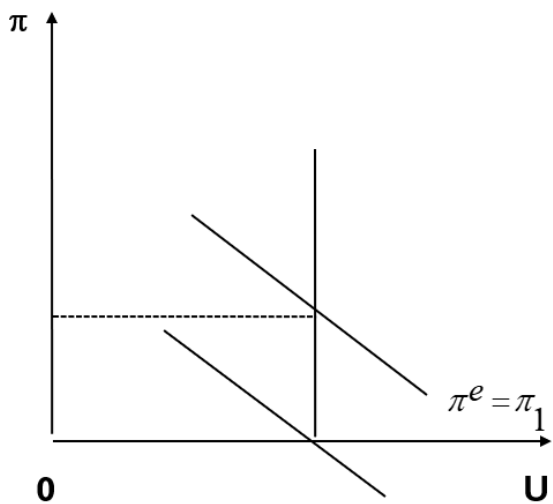
var U = var ε : no stabilisation.

More generally, Var U = $\left(\frac{1}{1+\beta a^2}\right)^2$ var $\varepsilon < 1$ depending on β .

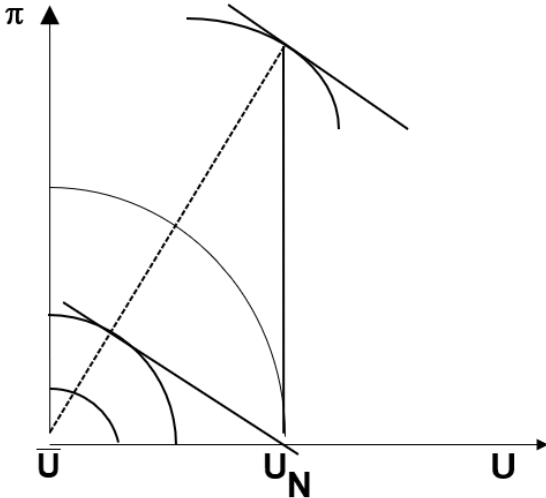
Indifference curves between inflation and excess unemployment:



Phillips curve with forward looking expectations:



Credibility of the monetary authority



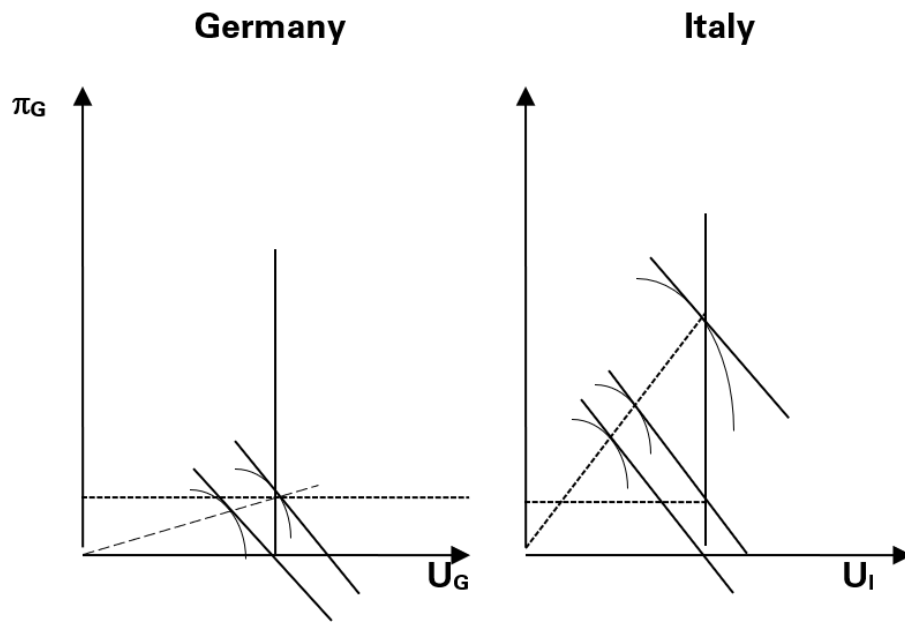
We use this model to analyse crisis

Assume PPP : $\dot{S} = \pi - \pi^*$ set $\pi^* = 0$

Fixing exchange rate is equivalent to announcing zero inflation

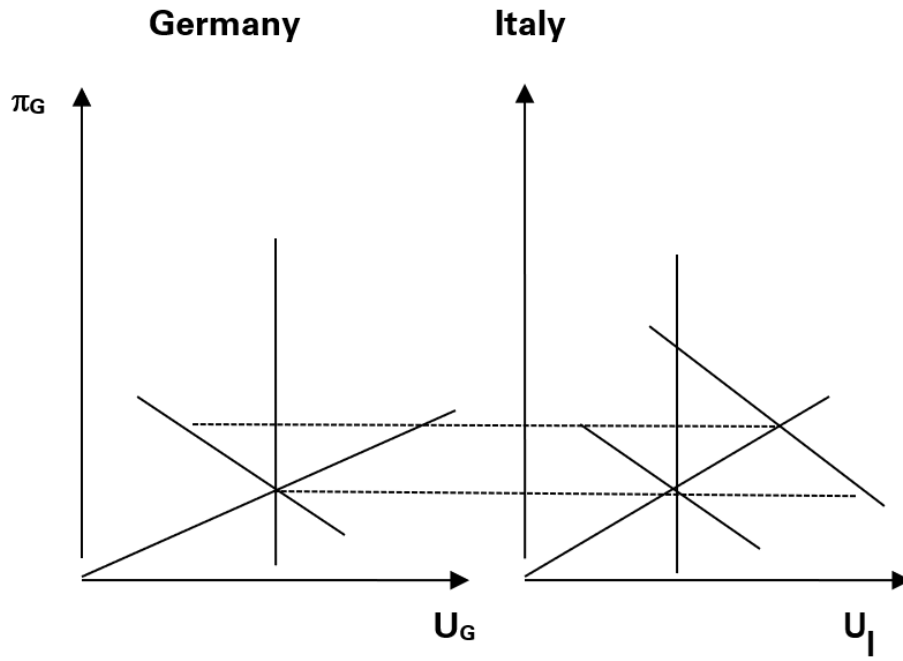
If there is no cost associated to devaluation fixing exchange rate is never credible

Italy will not follow its commitment to the announced FX



- If no cost of devaluation, the peg by Italy is not credible and will be attacked
- Two countries must have same preferences
- Even if they have same preferences, asymmetric shocks may reduce credibility

Italy must now accept too high unemployment to maintain the FX



- Italy must now accept too high unemployment
 - Italian authorities would like Germany to accommodate
 - If Germany refuses
 - conflict
 - loss of credibility
 - Attack on the system
 - case study : EMS during 1992 - 1993
-
- If there is a cost of devaluation, the fixed exchange rate can be made credible

The model with multiple equilibria

We will compute the loss of the authorities under different expectations of private agents

1. Loss under discretion

This is the loss when CB follows discretionary policy which is fully expected

This is solution derived earlier

$$\pi = \pi^e = \beta a(1 - \lambda) U_N \tag{16}$$

$$U = U_N \tag{17}$$

$$\bar{U} = \lambda U_N \tag{18}$$

Substitute into loss function : $L = \pi^2 + \beta(U - \bar{U})^2$

$$L_{DIS} = \beta^2 a^2 [(1 - \lambda) U_N]^2 + \beta [U_N(1 - \lambda)]^2 \quad (19)$$

$$L_{DIS} = \beta [(1 - \lambda) U_N]^2 (\beta a^2 + 1) = \beta k^2 (1 + \beta a^2) \quad (20)$$

In[13]:= **LDIS = $\beta k^2 (1 + \beta a^2)$**

Out[13]=

$$(1 + \beta a^2) \beta k^2$$

2. Loss of fixed exchange rate (zero inflation) when agents expect fixed exchange rate

$$\pi = \pi^e = 0 \quad (21)$$

$$U = U_N \quad (22)$$

$$L_{0,\pi^e=0} = \beta [U_N - \bar{U}]^2 = \beta [(1 - \lambda) U_N]^2 = \beta k^2 \quad (23)$$

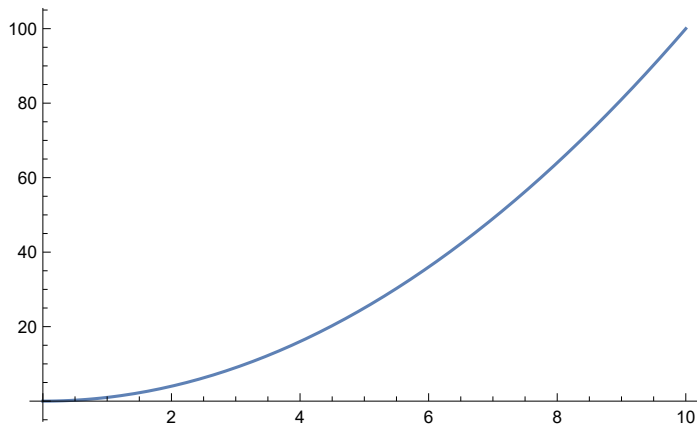
```
In[14]:= LFXZERO =  $\beta k^2$ 
```

```
Out[14]=
```

$$\beta k^2$$

```
In[15]:= Plot[LFXZERO, { $\beta k$ , 0, 10}]
```

```
Out[15]=
```



3. Loss when the CB devalues (cheats) while agents expect fixed exchange rate

Take optimal inflation from first order condition and set $p_e = 0$ (also $\epsilon = 0$)

$$\pi = \frac{\beta a(1-\lambda) U_N}{1+\beta a^2} + \frac{\beta a^2 \pi^e}{1+\beta a^2} + \frac{\beta a \varepsilon}{1+\beta a^2} \quad (24)$$

$$\pi = \frac{\beta a(1-\lambda) U_N}{1+\beta a^2} \quad (25)$$

$$U = U_N + a \left[-\frac{\beta a(1-\lambda) U_N}{1+\beta a^2} \right] \quad (26)$$

Substitute into loss function

$$\text{In[16]:= } \frac{\beta a (1 - \lambda) UN}{1 + \beta a^2}$$

$$\text{Out[16]= } \frac{UN \beta a (1 - \lambda)}{1 + \beta a^2}$$

$$\text{In[17]:= } UN + a \left[- \frac{UN \beta a (1 - \lambda)}{1 + \beta a^2} \right]$$

$$\text{Out[17]= } UN + a \left[- \frac{UN \beta a (1 - \lambda)}{1 + \beta a^2} \right]$$

$$\begin{aligned}
 L_{cheat} &= \left[\frac{\beta a(1-\lambda) U_N}{1+\beta a^2} \right]^2 + \beta \left[(1-\lambda) U_N - \frac{\beta a^2(1-\lambda) U_N}{1+\beta a^2} \right]^2 \\
 &= \left[\frac{\beta a(1-\lambda) U_N}{1+\beta a^2} \right]^2 + \beta \left[\frac{(1-\lambda) U_N}{1+\beta a^2} \right]^2 \quad (27) \\
 &= \frac{\beta [(1-\lambda) U_N]^2}{1+\beta a^2}
 \end{aligned}$$

$$\text{In[18]:= } (\beta a)^2 \left(\frac{(1-\lambda) UN}{1+\beta a^2} \right)^2 + \beta \left(\frac{(1-\lambda) UN}{1+\beta a^2} \right)^2$$

Out[18]=

$$\frac{UN^2 \beta (1-\lambda)^2}{(1+\beta a^2)^2} + \frac{UN^2 \beta a^2 (1-\lambda)^2}{(1+\beta a^2)^2}$$

$$\text{In[19]:= } \beta (\beta a^2 + 1)$$

Out[19]=

$$\beta [1 + a^2 \beta]$$

$$\text{In[20]:= } \% == (\beta a)^2 + \beta$$

Out[20]=

$$\beta [1 + a^2 \beta] == \beta + \beta a^2$$

$$\text{In[21]:= } \text{Equal}[\%]$$

Out[21]=

True

In[22]:= **LCHEAT** = $\frac{\beta k^2}{1 + \beta a^2}$

Out[22]= $\frac{k^2 \beta}{1 + a^2 \beta}$

The difference between LFXZERO and LCHEAT and measures the temptation to devalue

In[23]:= **LFXZERO - LCHEAT**

Out[23]=

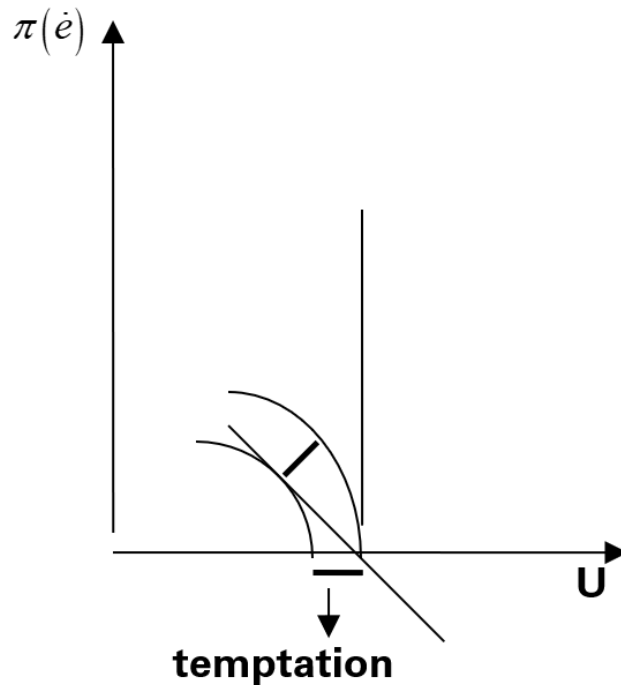
$$-\frac{k^2 \beta}{1 + a^2 \beta} + \beta k^2$$

In[24]:= **Together** $\left[-\frac{k^2 \beta}{1 + a^2 \beta} + \beta k^2 \right]$

Out[24]=

$$\frac{-k^2 \beta + \beta k^2 + a^2 \beta \beta k^2}{1 + a^2 \beta}$$

$$\begin{aligned} L_{0, \pi^e=0} - L_{cheat} &= \beta k^2 - \frac{\beta k^2}{1 + \beta a^2} \\ &= \beta k^2 \left(1 - \frac{1}{1 + \beta a^2} \right) \\ &= \frac{\beta^2 k^2 a^2}{1 + \beta a^2} > 0 \end{aligned} \tag{28}$$



Suppose there is a fixed cost of devaluation (e.g. it is costly politically; cf. R. Cooper)

Call this cost C

Then if $C > \text{temptation} = \frac{\beta^2 k^2 a^2}{1+\beta a^2}$ fixed exchange rate is credible

This does not yet mean, however, that it will last

4. Loss when authorities maintain fixed exchange rate while agents expect devaluation (discretion)

Call this loss under stabilisation

$$\pi = 0$$

$$\pi^e = \beta a(1 - \lambda) U_N = \beta a k \quad (29)$$

$$U = U_N + a(\beta a(1 - \lambda) U_N)$$

$$\begin{aligned} L_{stab} &= 0 + \beta \left[U_N + a(\beta a(1 - \lambda) U_N) - \bar{U} \right]^2 = \beta \left[(1 - \lambda) U_N + a^2 \beta (1 - \lambda) U_N \right]^2 \\ &= \beta \left[(1 - \lambda) U_N (1 + a^2 \beta) \right]^2 = \beta k^2 (1 + a^2 \beta)^2 \end{aligned} \quad (30)$$

In[25]:= $\beta \left((1 - \lambda) UN + a^2 \beta (1 - \lambda) UN \right)^2$

Out[25]=

$$\beta \left(UN (1 - \lambda) + a^2 UN \beta (1 - \lambda) \right)^2$$

In[26]:= **Collect** $\left[\beta \left(UN (1 - \lambda) + a^2 UN \beta (1 - \lambda) \right)^2, (1 + 2 a \beta) \right]$

Out[26]=

$$\beta \left(UN (1 - \lambda) + a^2 UN \beta (1 - \lambda) \right)^2$$

In[27]:= **Equal** $\left[UN (1 - \lambda) + a^2 UN \beta (1 - \lambda) == UN (1 - \lambda) (1 + a^2 \beta) \right]$

Out[27]=

True

In[28]:= **LSTAB** = $\beta k^2 (1 + a^2 \beta)^2$

Out[28]=

$$(1 + a^2 \beta)^2 \beta k^2$$

Comparing LDIS with LSTAB allows us to find the cost of defending the peg

$$\begin{aligned} L_{stab} - L_{dis} &= \beta k^2 (1 + a^2 \beta)^2 - \beta k^2 (1 + a^2 \beta) = \beta k^2 (1 + a^2 \beta) a^2 \beta \\ &= \beta^2 a^2 k^2 (1 + a^2 \beta) > 0 \end{aligned} \quad (31)$$

In[29]:= **LSTAB - LDIS**

Out[29]=

$$(1 + a^2 \beta)^2 \beta k^2 - (1 + \beta a^2) \beta k^2$$

In[30]:= **Simplify** $\left[(1 + a^2 \beta)^2 \beta k^2 - (1 + \beta a^2) \beta k^2 \right]$

Out[30]=

$$\left(-1 + (1 + a^2 \beta)^2 - \beta a^2 \right) \beta k^2$$

In[31]:= **Equal** $\left[\left(-1 + (1 + a^2 \beta)^2 - \beta a^2 \right) \beta k^2 == \beta k^2 (1 + \beta a^2) a^2 \beta \right]$

Out[31]=

True

In[32]:= $(1 + a^2 \beta)^2$

Out[32]=

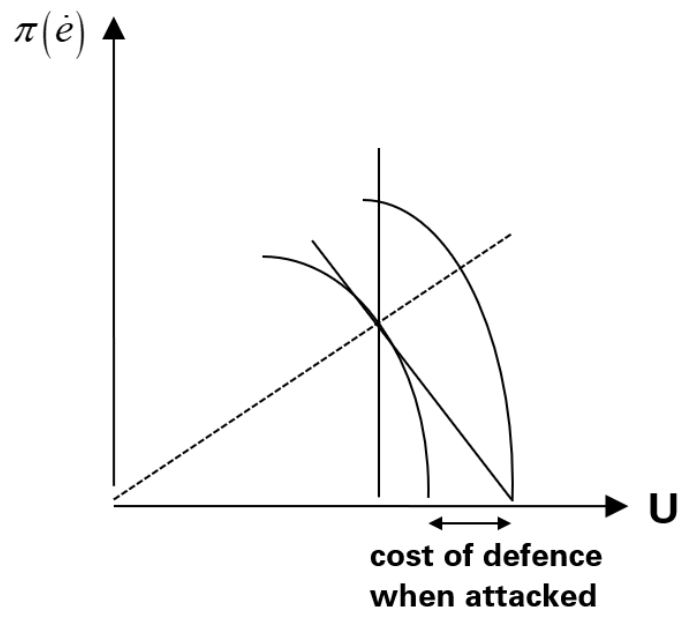
$$(1 + a^2 \beta)^2$$

In[33]:= **Expand** $\left[(1 + a^2 \beta)^2 \right] == 1 + 2 a^2 \beta + (a^2 \beta * a^2 \beta)$

Out[33]=

True

Cost of defending when attacked > Temptation when no attack



If $C > \text{cost of defence} = \beta^2 a^2 k^2 (1 + a^2 \beta)$ authorities will not yield

Speculators know this

Therefore they will not attack

Three possible cases

1. $\frac{\beta^2 k^2 a^2}{1+a^2\beta} < \beta^2 k^2 a^2(1+a^2\beta) < C$

Temptation < cost of defence when attacked < C

Fixed exchange rate is credible and will not be attacked

$$2. C < \frac{\beta^2 k^2 a^2}{1+a^2 \beta} < \beta^2 k^2 a^2 (1 + a^2 \beta)$$

Fixed exchange rate has no credibility and will be attacked with certainty

3. $\frac{\beta^2 k^2 a^2}{1+a^2\beta} < C < \beta^2 k^2 a^2(1 + a^2\beta)$

Two expectations are model consistent:

- Agents do not expect a devaluation;

In that case authorities' temptation to devalue is smaller than the cost of devaluation

So, authorities do not devalue

- Agents expect a devaluation;

In this case the authorities' cost of defending the peg exceeds the cost of devaluation

So, authorities devalue

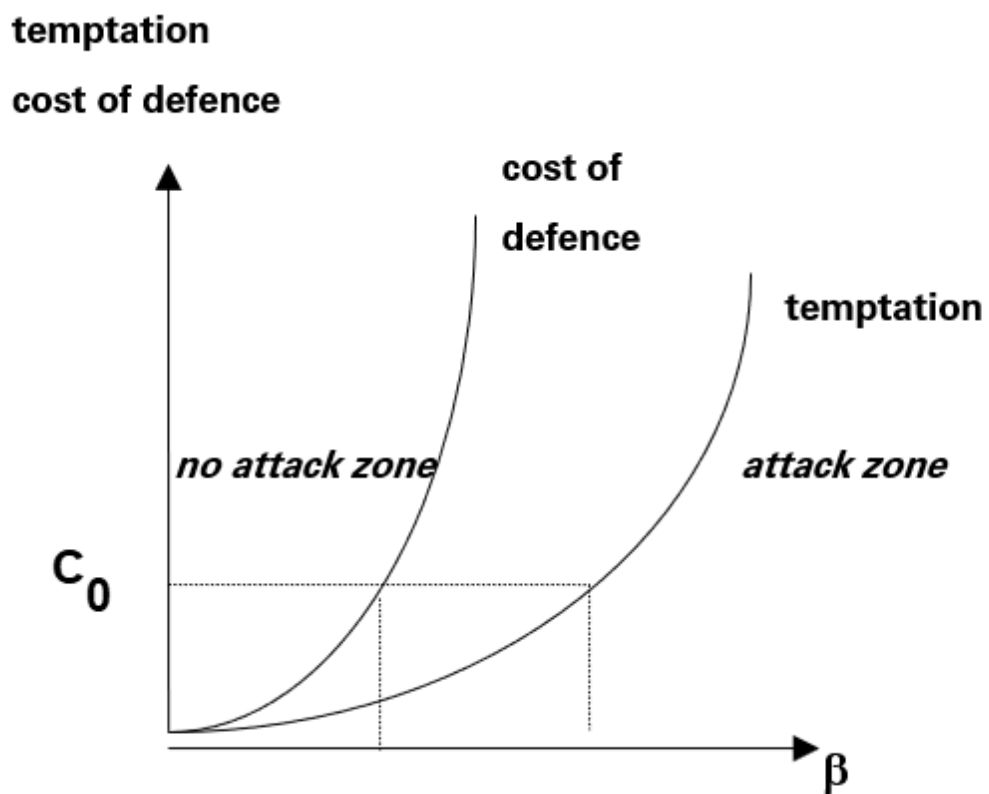
There are two possible equilibria

Which one will prevail solely depends on expectations which become self - fulfilling

Importance of weight attached to stabilisation (β)

Recall that $\frac{a^2 \beta^2 k^2}{1+a^2 \beta}$: Temptation to devalue

Recall that $a^2 \beta^2 k^2 (1 + a^2 \beta)$: Cost of defence when attacked



- If $\beta = 0$ (no desire to stabilize) then $C > 0$ ensures there is never an attack
- With cost of devaluation C_0
 - \Rightarrow range of β between β_1 and β_2 with multiple equilibria
- With β sufficiently \Rightarrow high always attack

Two conclusions

- 1) Too strong ambitions to stabilise make fixed exchange rate regime fragile
- 2) Making devaluation more costly, improves conditions for survival of fixed exchange rate

Case study : ERM during 1996 - 1998

cost of devaluation was drastically increased (cf. convergence criteria)